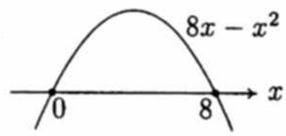
Lösungsvorschlag zu Grundaufgaben zur ln-Funktion

Nr		BE

9.1 $f(x) = \ln(8x - x^2)$, $\quad D_f$: $8x - x^2 = x(8-x) = 0$: $\quad x_1 = 0$, $\quad x_2 = 8$

$$\Longrightarrow D_f =]\,0; 8\,[$$



NSt.: $\ln(8x - x^2) = 0 \iff 8x - x^2 = 1 \iff x^2 - 8x + 1 = 0$

$$x_{1,2} = \frac{8 \pm \sqrt{64-4}}{2} = \frac{8 \pm 2\sqrt{15}}{2} = 4 \pm \sqrt{15}$$

$$\lim_{\substack{x\to 0\\ x>0}} \ln(8x - x^2) = \text{„}\ln(+0)\text{"} = -\infty \ ; \quad \lim_{\substack{x\to 8\\ x<8}} \ln(8x - x^2) = \text{„}\ln(+0)\text{"} = -\infty$$

$\Longrightarrow x = 0$ und $x = 8$ vert. Asymptoten von G_f

9.2 $f'(x) = \dfrac{8-2x}{8x - x^2}$

$$f''(x) = \frac{(8x-x^2)\cdot(-2) - (8-2x)(8-2x)}{(8x-x^2)^2} = \frac{-16x + 2x^2 - (64 - 32x + 4x^2)}{(8x-x^2)^2} =$$

$$= \frac{-2(x^2 - 8x + 32)}{(8x-x^2)^2}$$

9.3 **Monotonie:** $f'(x) = 0$: $\quad 8 - 2x = 0 \iff x = 4$, $\quad f(4) = \ln(16) = \ln(2^4) = 4\ln(2)$

D_f:	0	4	8
$8-2x$:	+	−	
$8x-x^2$:	+	+	
$f'(x)$:	+	−	
		HOP	

f streng mon. zunehmend in $]0; 4]$

f streng mon. abnehmend in $[4; 8[$

$\Longrightarrow H(4|4\ln(2))$ Hochpunkt von G_f

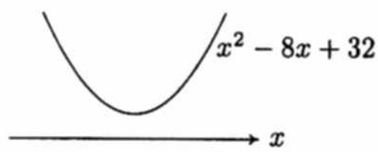
9.4 **Krümmung:** $f''(x) = 0$: $\quad x^2 - 8x + 32 = 0$,

Diskriminante $D = 64 - 128 < 0$

$\Longrightarrow x^2 - 8x + 32 > 0$ in D_f

da Nenner von $f''(x) > 0 \Longrightarrow f''(x) < 0$ in D_f

$\Longrightarrow G_f$ rechtsgekrümmt in $D_f \Longrightarrow$ kein Wendepunkt



9.5